Renaissance College
Mathematics Deparment
Adapted from the Santiago College

Sample Investigation (Criteria B & C)

Given fact

Using matchsticks, we can form the triangles as shown below.

The goal

Determine the relationship between the number of matchsticks and the number of triangles.

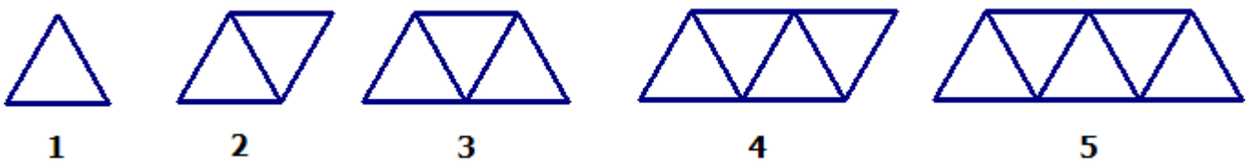
Development of the investigation

1. To define the problem and goal

   In this investigation, I will need to determine the relationship between the number of matchsticks and the number of triangles formed. With this rule I can figure out the number of matchsticks needed to draw any number of triangles.

2. To gather more data

   I will draw the next few diagrams and see if there are any patterns. The number below the diagram indicates both the diagram number and the number of triangles.



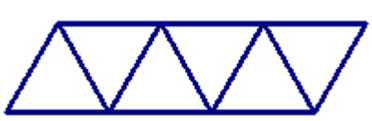
3. To organize results in a table and comment on the pattern(s) found

   From my drawings above I have found that I need to add two matchsticks whenever I increase a triangle. I will organize my results in a table and see if further patterns can be found.

Number of triangles (diagram number)	Number of the matchsticks
1	3 = 1+2
2	5 = 3+2 = 1+2+2
3	7 = 3+2+2 = 1+2+2+2
4	9 = 3+2+2+2 = 1+2+2+2+2
5	11 = 3+2+2+2+2 = 1+2+2+2+2+2

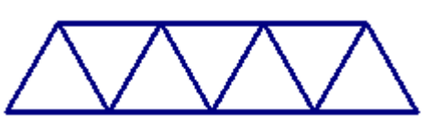
4. To raise and to prove conjecture

   Based on the table above, in order to construct 6 triangles, I will need 13 matchsticks or 1 + 2 + 2 + 2 + 2 + 2 + 2. The drawing below can verify my prediction



There are 13 matchsticks altogether.

In order to construct 7 triangles, I will need 15 matchsticks or 1 + 2 + 2 + 2 + 2 + 2 + 2 + 2. Verifying my prediction by using a drawing.



There are 15 matchsticks altogether.

As shown in the two cases above, the same pattern applies to diagram 6 and 7.

5. To find a general rule

Number of triangles (diagram number)	Number of the matchsticks
1	$\mathbf{1}\ \underbrace{+2}_{\text{one triangle}}$
2	$\mathbf{1}\ \underbrace{+2+2}_{\text{two triangles}}$
3	$\mathbf{1}\ \underbrace{+2+2+2}_{\text{three triangles}}$
4	$\mathbf{1}\ \underbrace{+2+2+2+2}_{\text{four triangles}}$
5	$\mathbf{1}\ \underbrace{+2+2+2+2+2}_{\text{five triangles}}$
	$1+2\times$ Number of triangles

According to the table, the rule can be determined as:

Total number of matchsticks = 2 × Number of triangles + 1

If ***N*** to be the total number of matchsticks needed to construct any number of triangles and ***n*** is the number of triangles, the general rule is $N = 2n + 1$.

6. To verify the general rule.

If I wanted to construct 10 triangles ($n = 10$), I will have to use $N = 2(10) + 1 = 21$ matchsticks. This can be verified with the diagram below.

There are 21 matchsticks.

If I wanted to construct 16 triangles ($n = 16$), I will have to use $N = 2(16) + 1 = 33$ matchsticks. This can be verified with the diagram below.

There are 33 matchsticks.

7. Justifications of the results.

Starting from one matchstick I need to add two matchsticks to form the first triangle.

That's why in the rule, $N = 2n + 1$, the 1 is constant in all the diagrams. This can also be seen in the tables above.

When an additional triangle is required, 2 matchsticks are added.

That's why in the rule, $N = 2n + 1$, the $2n$ is a variable term as the number of triangles is determined by the number of matchsticks that are added. Therefore, this justifies that my general rule is correct.